A Bound on Tableau Stabilization Using Lattice Paths

Jacob David, Chris Wu, Suho Oh, Connor Ahlbach

Abstract

If one attaches copies of a skew tableau to the right of itself by concatenating corresponding rows, after some point the entries only experience horizontal displacement under rectification, a phenomenon called tableau stabilization. Our purpose is to improve the original upper bound on the stabilization function to the number of rows of the skew tableau.

1. Introduction

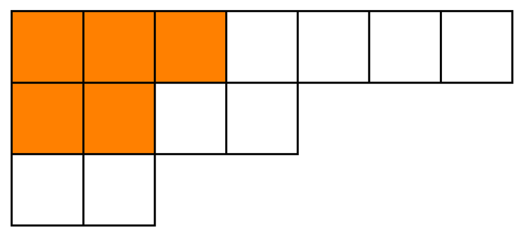


Figure 1: Example of a skew shape.

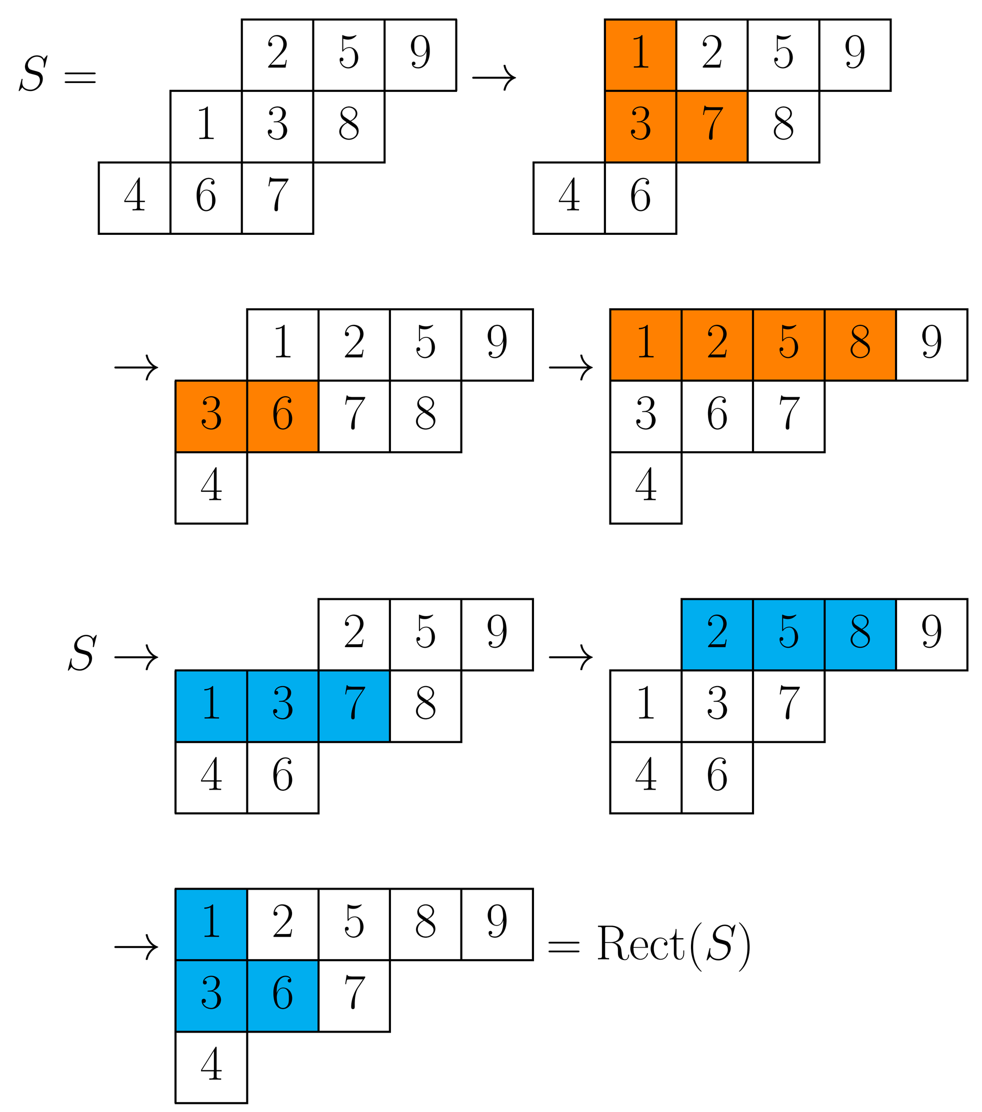


Figure 2: Rectification of SSYT S in two ways.

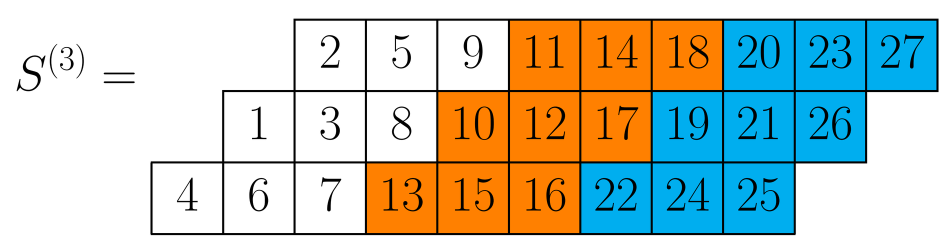


Figure 3: $S^{(3)}$ using the same S.

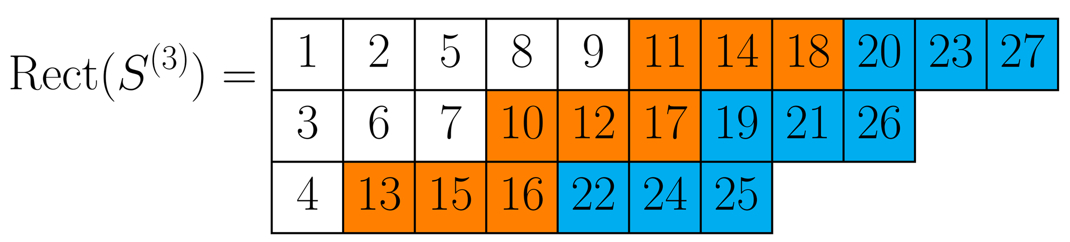


Figure 4: S stabilizes at 2 and 3, but not 1 $\implies \mathrm{stab}(S) = 2$.

Introduction (con't)

Goal: to improve the original upper bound of $\mathrm{stab}(T)$ to r, the number of rows.

2. Method

Theorem (Greene's Theorem)

Let π be a permutation and let $\ell_k(\pi)$ denote the maximum combined length of k disjoint increasing subsequences of π. For a non-skew tableau S, let $\mathrm{sh}(S)$ denote its shape. Then,

$$\ell_k(\pi) = \mathrm{sh}(P(\pi))_1 + \cdots + \mathrm{sh}(P(\pi))_k.$$

$\mathrm{Rect}(T) = P(w)$, where w is the reading word of SSYT T.

Corollary

Let π be the reading word of a (skew) standard Young tableau T and let $\ell_k(\pi)$ denote the maximum combined length of k disjoint increasing subsequences of π. For a non-skew tableau S, let $\mathrm{sh}(S)$ denote its shape. Then,

$$\ell_k(\pi) = \mathrm{sh}(\mathrm{Rect}(T))_1 + \cdots + \mathrm{sh}(\mathrm{Rect}(T))_k.$$

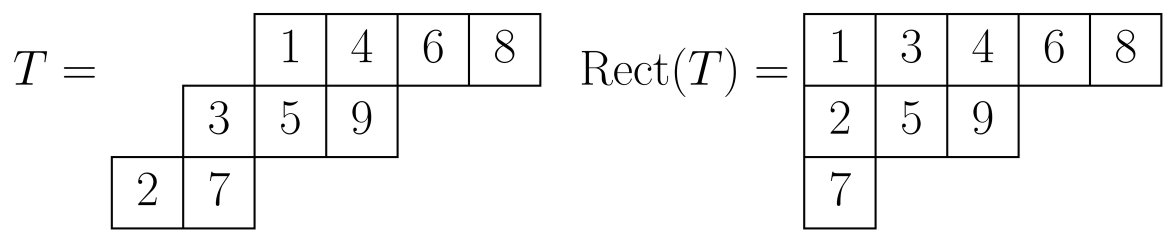


Figure 5: Example for Corollary:
$\pi = 273581468, \ell_1(\pi) = 5, \ell_2(\pi) = 8, \ell_3(\pi) = 9$

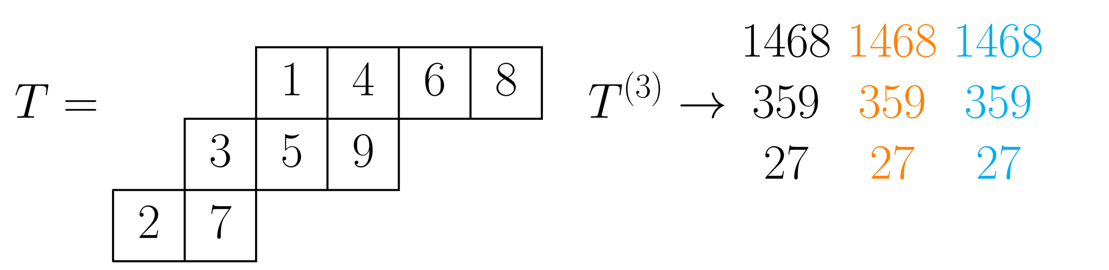


Figure 6: Matrix of Words.

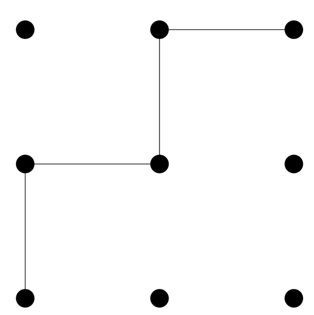


Figure 7: Lattice path 2735935914681468 plotted on a grid of lattice point.

Method (con't)

Any increasing subsequence w of the reading word of $T^{(k)}$ is a subsequence of a lattice path of M.

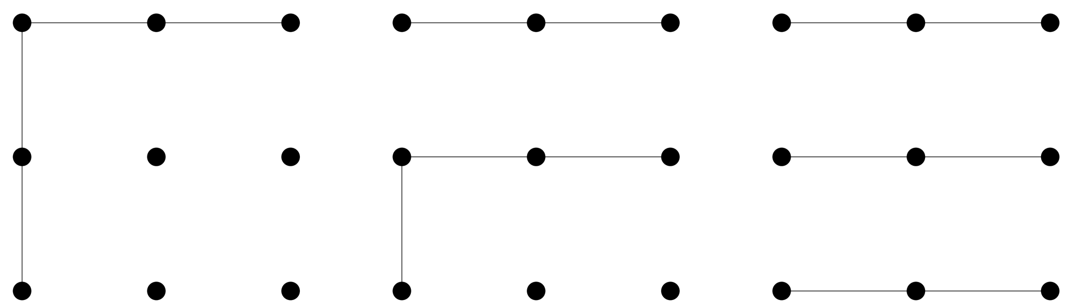


Figure 8: Sets of lattice paths that contain the subsequences that exhibit $\ell_1(\pi)$, $\ell_2(\pi)$, and $\ell_3(\pi)$, respectively, where π is the reading word of $T^{(3)}$.

Techniques

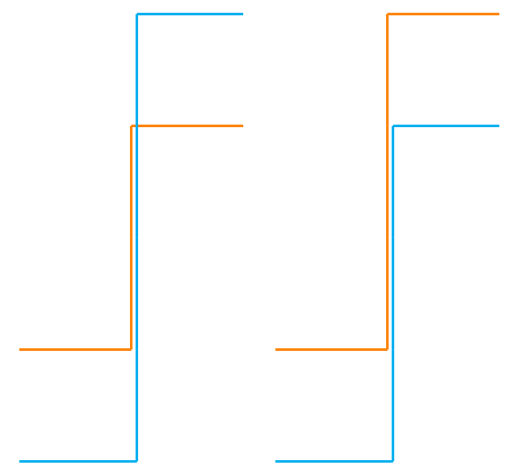


Figure 9: Top-down switching.

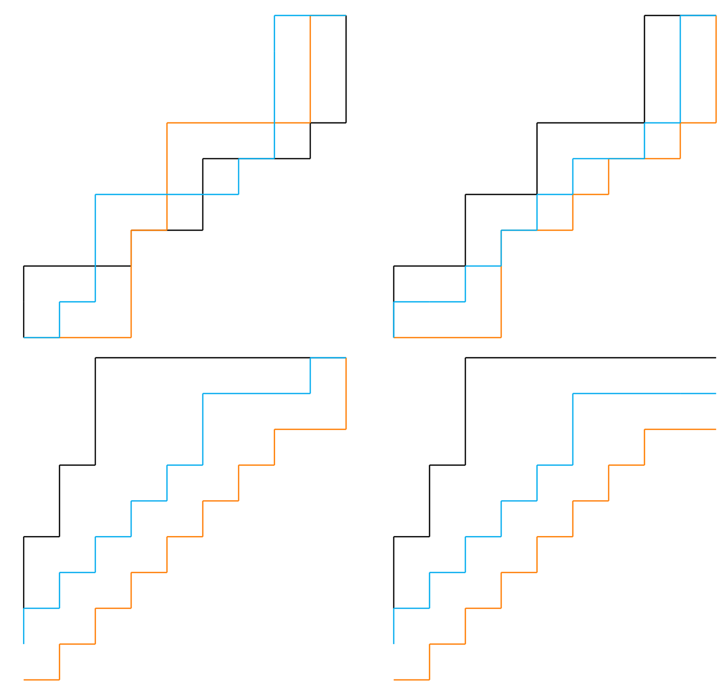


Figure 10: Techniques optimizing family of lattice paths.

Conclusion and Remarks

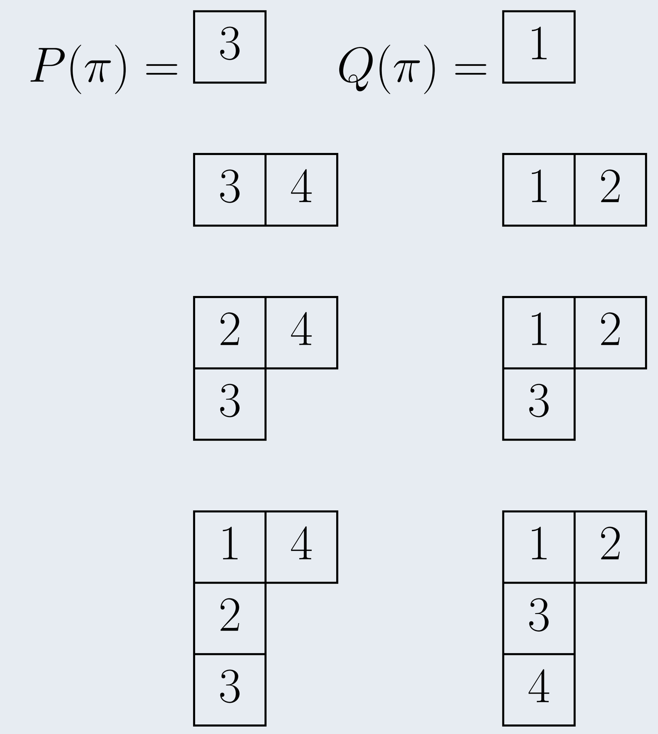
Main Result

For SSYT S with r rows of weakly decreasing sizes, $\mathrm{stab}(T) \leq r$.

- stab as a permutation statistic
- $Q(w) = Q(v) \implies \mathrm{stab}(w) = \mathrm{stab}(v)$

Notation and Terminology

- For SSYT S with weakly decreasing row sizes top to bottom, let $S^{(k)}$ denote the result of attaching $k-1$ shifted copies of S to the right of S so the result is standard.
- We say S *stabilizes* at k if the entries in the last copy of $S^{(k)}$ experience no vertical slides while rectifying. Let $\mathrm{stab}(S)$ denote the minimum value at which S stabilizes.
- The *reading word* of a tableau is the word obtained by concatenating the rows from bottom to top.
- The Robinson–Schensted–Knuth (RSK) correspondence is an algorithm that maps a permutation π of length n to a pair of Standard Young Tableaux (P, Q) of equal shape, each having n cells.
  Example: $\pi = 3421$:



- A *lattice path* within matrix M is a concatenated sequence of **entries of M** that move adjacently right or up at each step.

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