

# MAXIMAL GREEN SEQUENCES for TRIANGLE PRODUCTS

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# REDDENING SEQUENCES

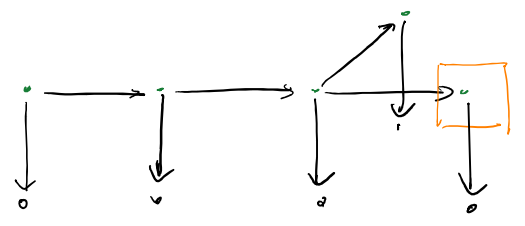
fitness notion for cluster

U1

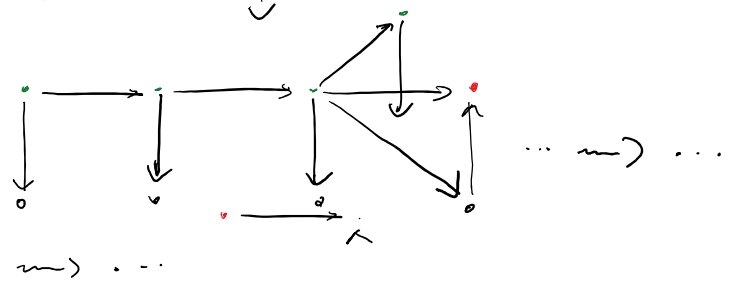
## MAXIMAL GREEN SEQUENCES

(BERNHARD KELLER)

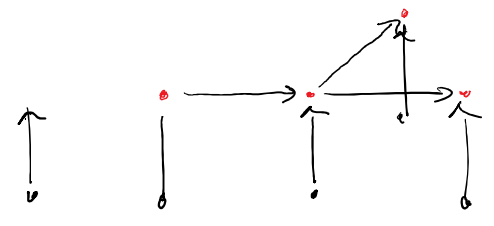
PRINCIPAL EXTENSION  $\tilde{Q}$ :



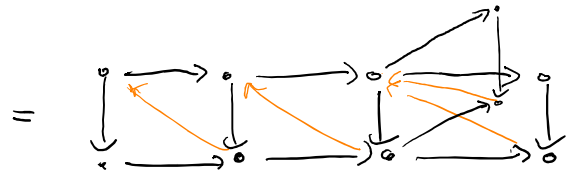
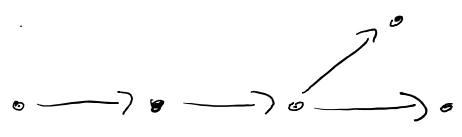
Mutation



CO-PRINCIPAL EXTENSION  $\check{Q}$



## TRIANGLE PRODUCT



product + "triangulation"

Q has source MGS S  $\Leftrightarrow$  Q acyclic

R has sink MGS T  $\stackrel{??}{\Leftrightarrow}$  R Dynkin

$\Rightarrow$   $Q \boxtimes R$  has source-sink MGS

$$S \boxtimes T = (s_1, t_1), \dots, (s_m, t_m), (s_1, t_2), \dots, (s_m, t_2), \dots, (s_m, t_m)$$

Application Fock-Goncharov conjecture holds for Q if  
(Gross-Hacking-Katz-Kontsevich)

i)  $\exists$  reddening sequence for Q

ii) every frozen vertex has optimized seed

iii) B-matrix of Q has full rank

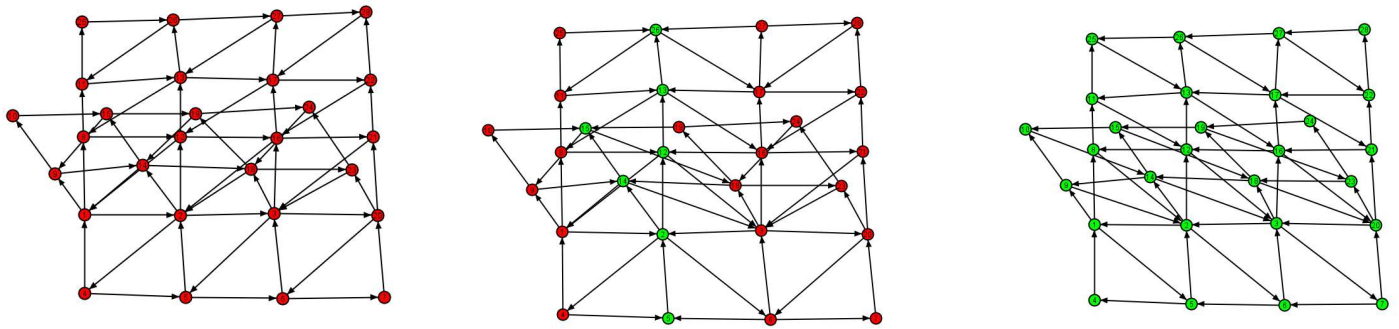
$\rightarrow \forall v$  frozen  $\exists s : v \in \hat{Q}^s$  sink

Dynkin  $\boxtimes A_n$   $\xrightarrow{\text{source-sink sequence}}$

Dynkin  $\boxtimes A_n \xrightarrow{\text{explicit optimization sequences}}$

$\rightsquigarrow$  i) - iii) hold for large double-bracket cells

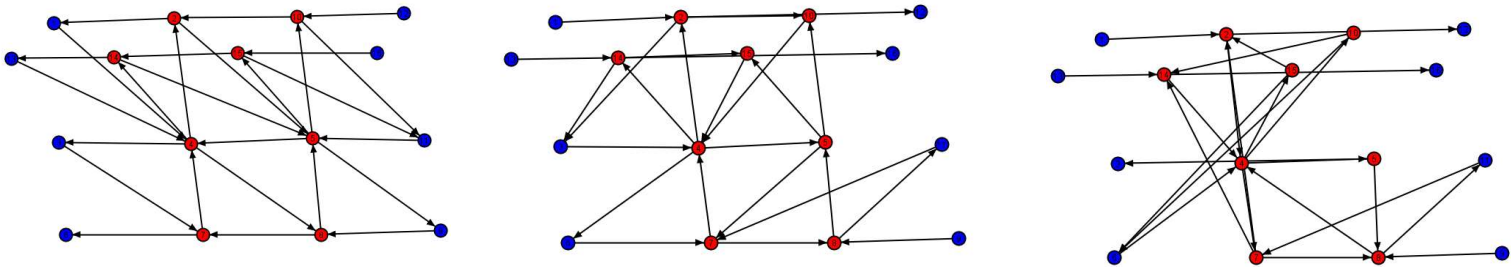
# SINK-SOURCE MAS FOR ACYCLIC $\boxtimes$ DYNKIN



4,1,8,11,25,9,10

5,2,12,13,26,14,15,6,3,16,17,27,18,19,7,20,21,22,28,23,24,  
 4,1,8,11,25,9,10,5,2,12,13,26,14,15,6,3,16,17,27,18,19,  
 4,1,8,11,25,9,10,5,2,12,13,26,14,15,  
 4,1,8,11,25,9,10

# Optimization SEQUENCE for $D_4$ DOUBLE-BRUTAL CELL



15,10,5,8,14,2,4,7,15,10,5,8

4,5