Cell Complexes, Poset Topology and the Representation Theory of Algebras Arising in Algebraic Combinatorics and Discrete Geometry

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Franco Saliola, Université du Québec à Montréal
Benjamin Steinberg, City College of New York

FPSAC 2021 Bar-Ilan University, January 10, 2021

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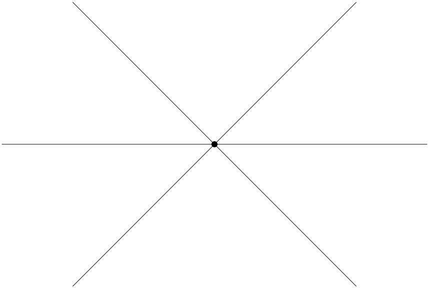
a set of hyperplanes partitions $\mathbb{R}^n$ into *faces*:



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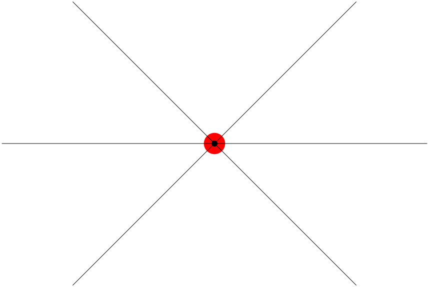

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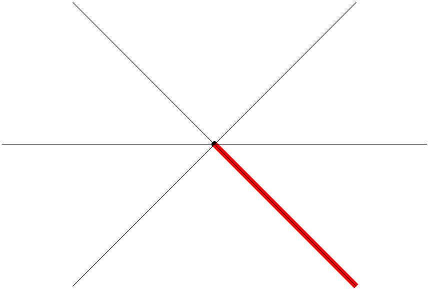

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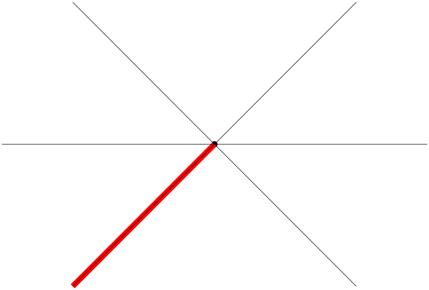

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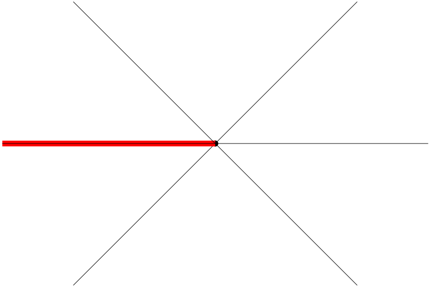

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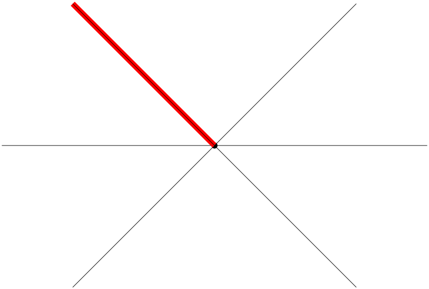

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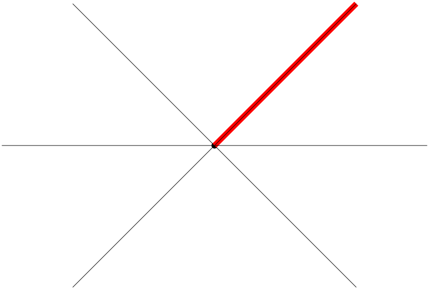

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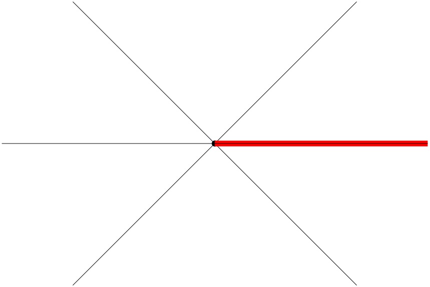

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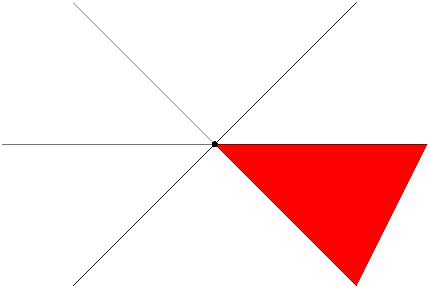

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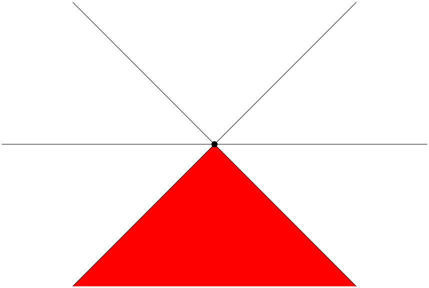

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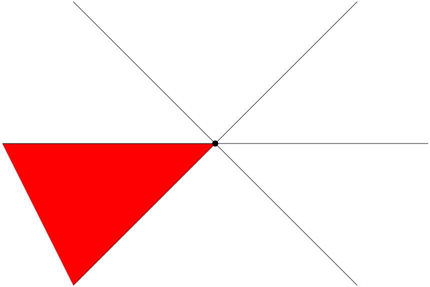

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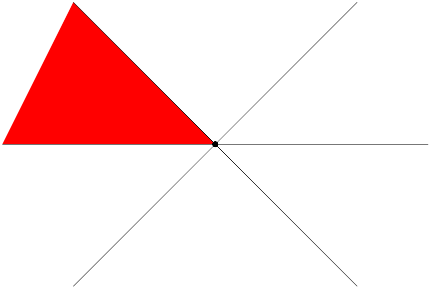

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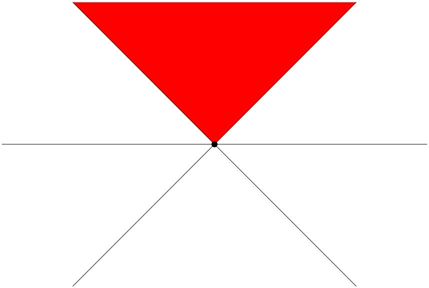

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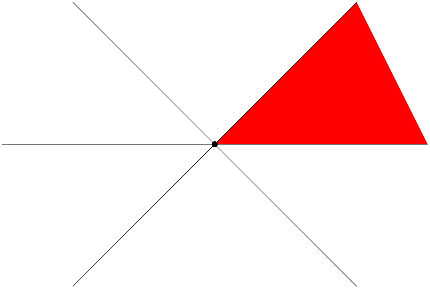

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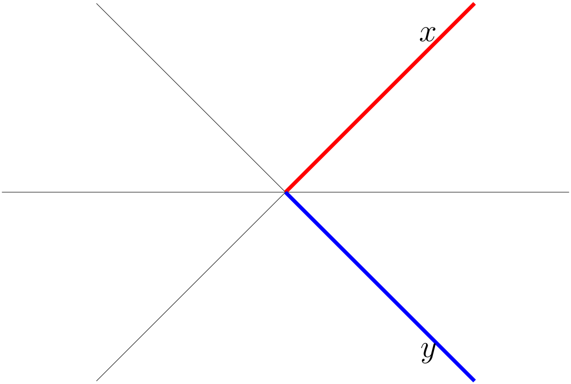

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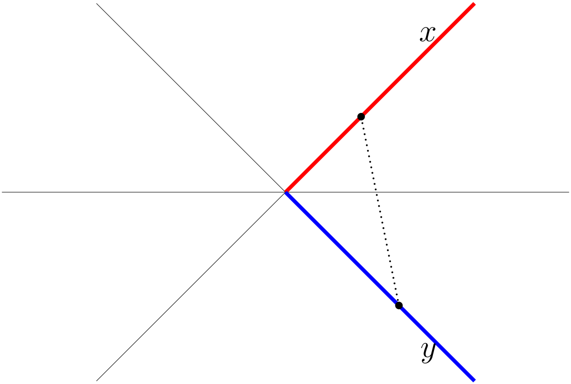

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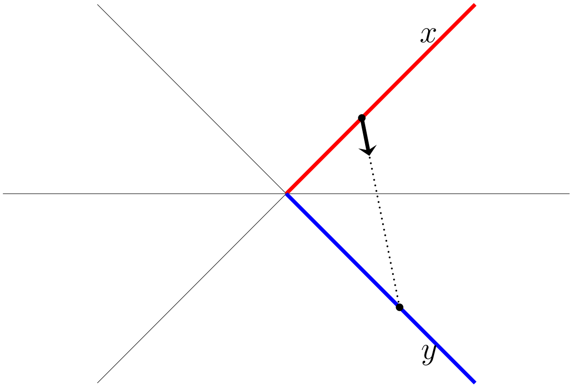

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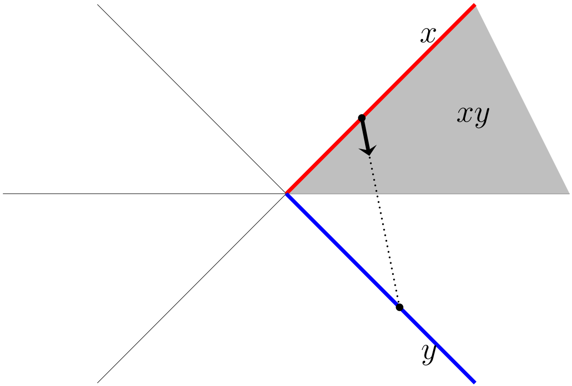


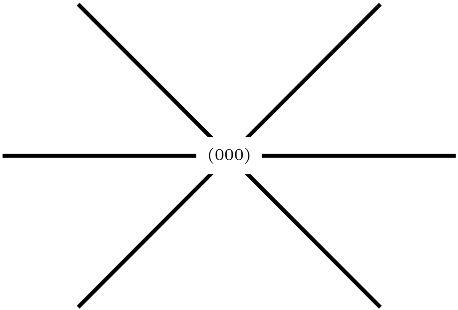
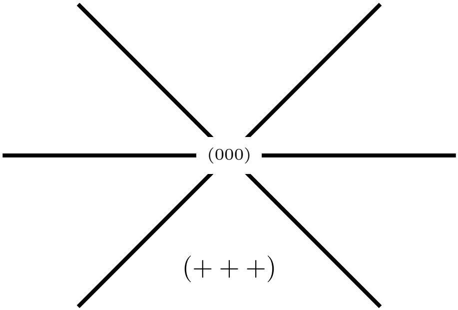


Figure: The sign sequences of the faces of the hyperplane arrangement in $\mathbb{R}^2$ consisting of three distinct lines.



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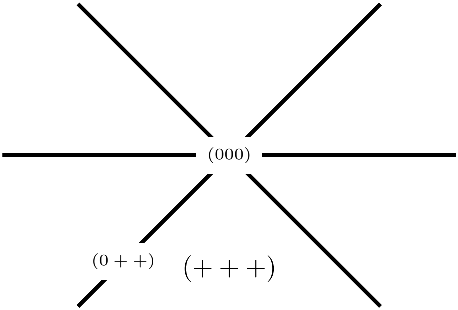

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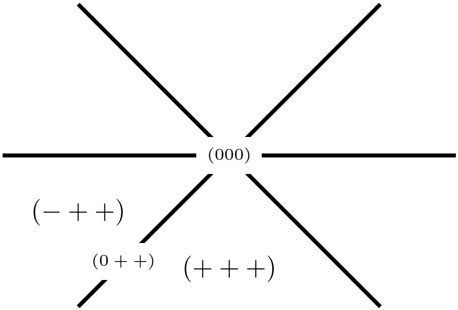

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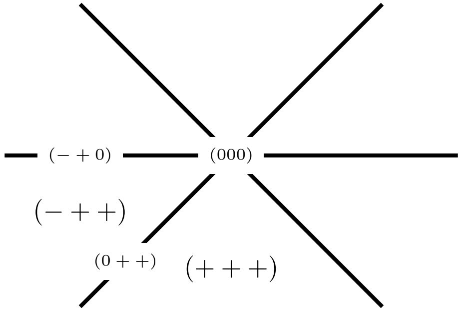

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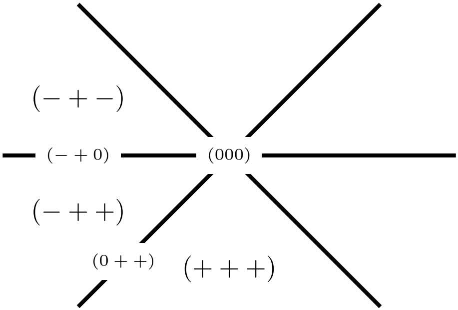

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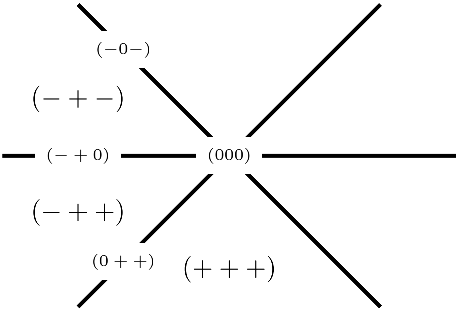

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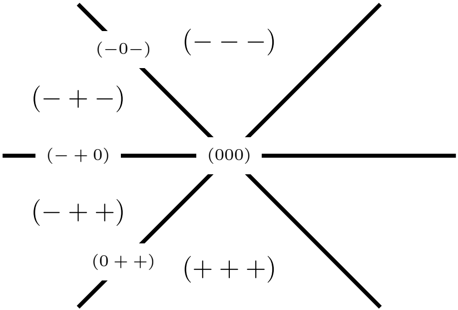

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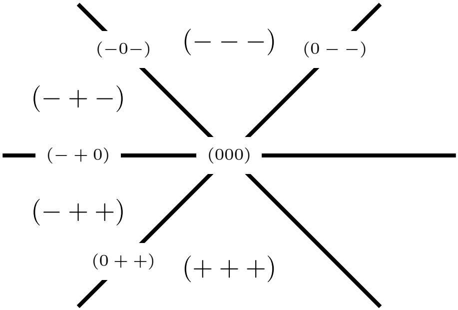

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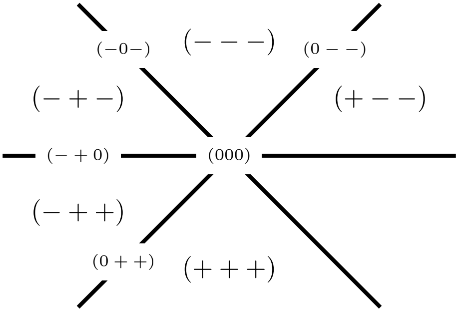

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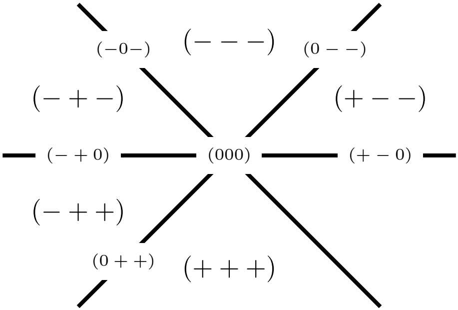

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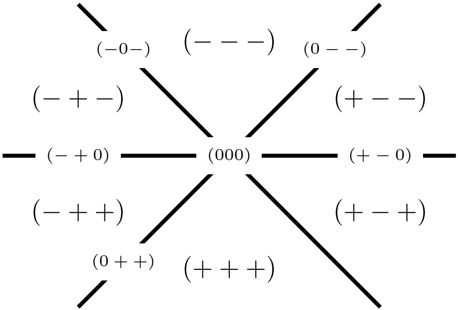

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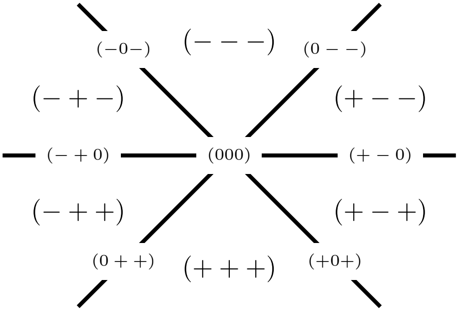

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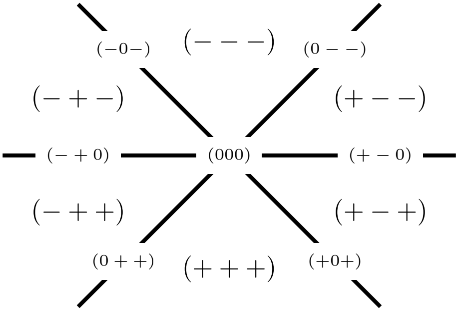

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The geometric product is just multiplication in the monoid $\{0, +, -\}^3$, where 0 is the identity element and $++=+$-$=+$, --$=$-$+=$-.

Left-regular bands (LRBs)

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Remarks

- *Informally: identities say ignore "repetitions".*
- *We consider only finite monoids here.*

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$\Delta((B, \leq))$ is contractible, since 1 is a cone point.

Representation Theory of LRBs

- Simple $\mathbb{K}B$-modules and its Jacobson Radical

Let $\Lambda(B)$ denote the lattice of principal left ideals of B, ordered by inclusion:

$$\Lambda(B) = \{Bb : b \in B\} \qquad Ba \cap Bb = B(ab)$$

Monoid surjection:

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where $\overline{\sigma} : \mathbb{K}B \rightarrow \mathbb{K}(\Lambda(B))$ is the extended morphism. $\mathbb{K}(\Lambda(B))$ is semisimple and so simple $\mathbb{K}B$-modules S_X are indexed by $X \in \Lambda(B)$.

Semisimple Quotient and Simple Modules

$$\mathbb{K}B/\operatorname{rad}(\mathbb{K}B) \cong \mathbb{K}B/\ker(\overline{\sigma}) \cong \mathbb{K}\Lambda(B) \cong \mathbb{K}^{\Lambda(B)}$$

For each $X \in \Lambda(B)$, the corresponding simple module is 1 dimensional and is given by the following action.

$$\rho_X(a) = \begin{cases} 1, & \text{if } \sigma(a) \geq X, \\ 0, & \text{otherwise} \end{cases}$$

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We see then that $\mathbb{K}B$ is a basic algebra: All of its simple modules are 1 dimensional. Equivalently, $\mathbb{K}B$ has a faithful representation by triangular matrices.

Basic Algebras

Let $\mathbb{K}$ be an algebraically closed field.

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The following conditions are equivalent.

1. A *is a finite dimensional basic algebra over* $\mathbb{K}$.
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Every finite dimensional algebra over $\mathbb{K}$ *is Morita equivalent to a unique basic algebra.*

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- This has been done by: Bidigare, Hanlon and Rockmore; Diaconis and Brown; Brown; Björner; Diaconis and Athanasiadis; and Chung and Graham.

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- ordering of the books $\leftrightarrow$ word containing every letter

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Tsetlin Library: shelf of books

"use a book, then put it at the front"

- ordering of the books $\leftrightarrow$ word containing every letter
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- long-term behavior: favorite books move to the front

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Others:

Björner, Athanasiadis-Diaconis, Chung-Graham, ...

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The *free partially-commutative LRB* $F(G)$ on a graph $G = (V, E)$ is the LRB with presentation:

$$F(G) = \left\langle V \;\middle|\; xy = yx \text{ for all edges } \{x, y\} \in E \right\rangle$$

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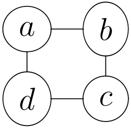
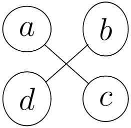
- LRB-version of the Cartier-Foata *free partially-commutative monoid* (aka *trace monoids*).

Acyclic orientations

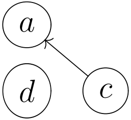
Elements of $F(G)$ correspond to acyclic orientations of induced subgraphs of the complement $\overline{G}$.

Example

$G =$ 

$\overline{G} =$ 

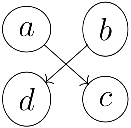
Acyclic orientation on induced subgraph on vertices $\{a, d, c\}$:



In $F(G)$: $cad = cda = dca$ (c comes before a since $c \to a$)

Random walk on $F(G)$

States: acyclic orientations of the complement $\overline{G}$



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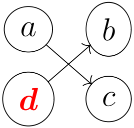

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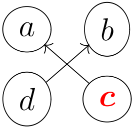

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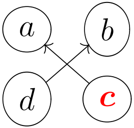

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Athanasiadis-Diaconis (2010): studied this chain using a different LRB (graphical arrangement of G)

The (Karnofsky)-Rhodes Expansion of a Semilattice

If Λ is a semilattice let $\Delta(\Lambda) = \{x_1 > x_2 \ldots > x_k | x_i \in \Lambda\}$ be the set of chains in Λ.

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- It is an LRB whose $\mathcal{R}$ order has Hasse diagram a tree and $\mathcal{L}$ order is the Hasse diagram of Λ.

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$$31245 \leftrightarrow \emptyset < \{3\} < \{3,1\} < \{3,1,2\} < \{3,1,2,4\} < \{3,1,2,4,5\}$$

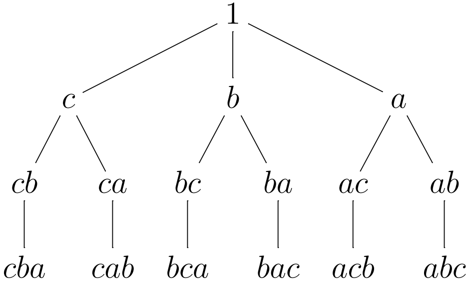
- Many of the LRBs on combinatorial structures are submonoids of (Karnofsky)-Rhodes expansions of semilattices.

Poset of a LRB

B is a partially-ordered set via its $\mathcal{R}$-order:

$$a \le b \quad \Leftrightarrow \quad ba = a$$

Example: $F(\{a, b, c\})$

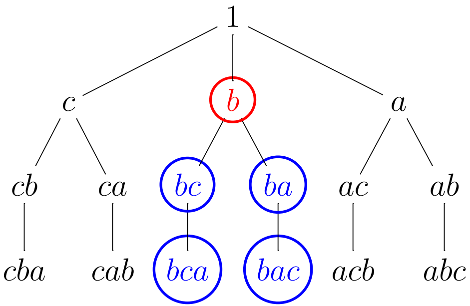


Certain subposets of a LRB

For $Ba \subseteq Bb$, consider the subposet of B:

$$B_{[Ba,Bb)} = \left\{ x \in B : x < b \text{ and } Ba \leq Bx \right\}$$

Example: $B(abc) \subseteq Bb$



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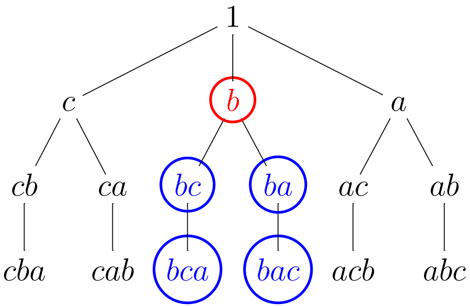

$$B_{[Babc,Bb)} = \{bc, ba, bca, bac\}$$

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where $\Delta B_{[X,Y)}$ is the *order complex* of the subposet $B_{[X,Y)}$. This is the simplicial complex whose simplices are the chains (ordered subsets) of the poset.

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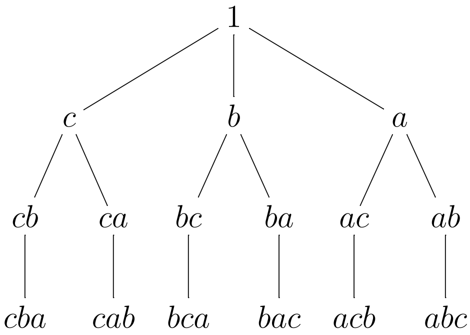
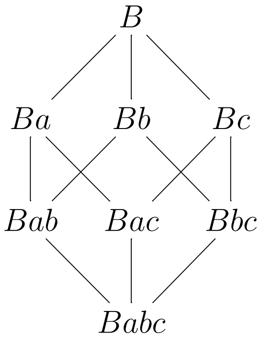
Poset and $\Lambda(B)$ for $B = F(\{a, b, c\})$





Quiver of $\mathbb{K}B$

The *(Ext)-quiver* of an algebra A is the digraph Q_A with:

- vertex set the simple A-modules S_X
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Theorem

Let A be a basic finite dimensional algebra. Then A is a quotient of the path algebra $P = \mathbb{K}Q_A$ of its quiver Q_A by an ideal I such that $(P^+)^n \subseteq I \subseteq (P^+)^2$, for some $n \geq 2$, where (P^+) is the ideal of positive length paths. Conversely, every such algebra is a finite dimensional basic algebra.

Corollary. Let B be a finite LRB. The quiver of $\mathbb{K}B$ has vertex set $\Lambda(B)$. The number of arrows $X \to Y$ is 0 if $X \not< Y$; otherwise, it is one less than the number of connected components of $\Delta B_{[X,Y)}$.

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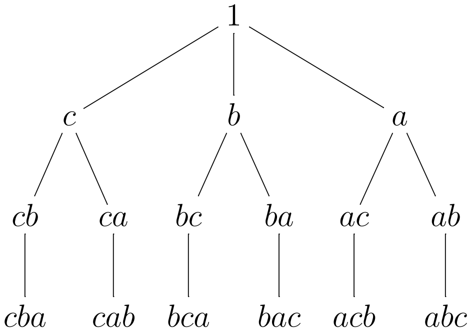
Proof. For $X < Y$:

$$\mathrm{Ext}^1_{\mathbb{K}B}(S_X, S_Y) = \widetilde{H}^0\big(\Delta B_{[X,Y)}, \mathbb{K}\big)$$

Computing the quiver of $B = F(\{a, b, c\})$



B

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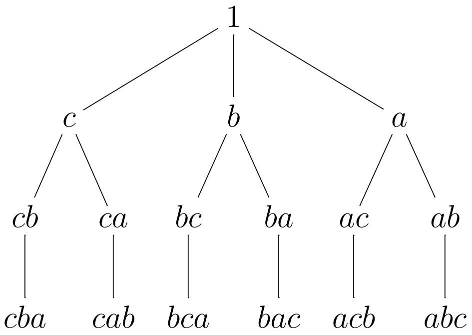

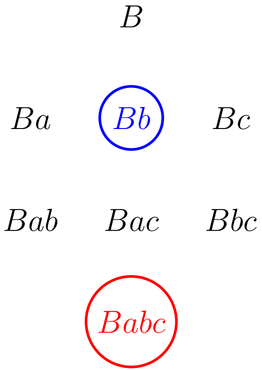

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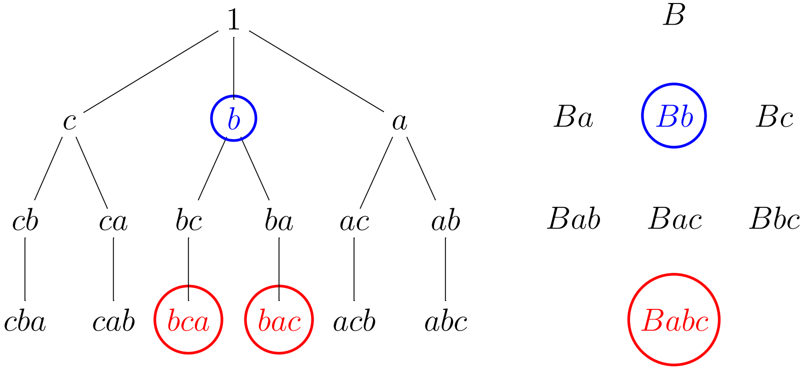

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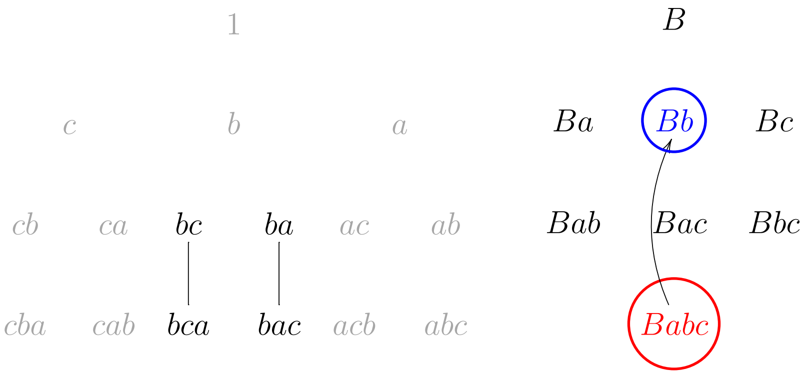

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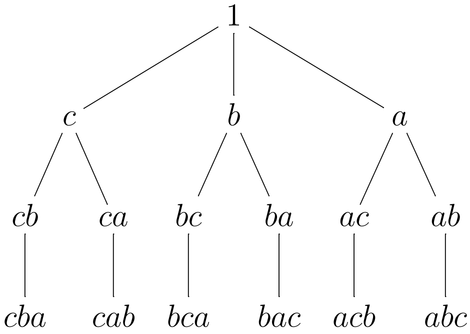

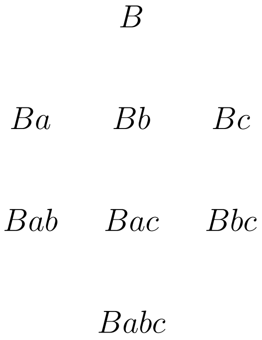

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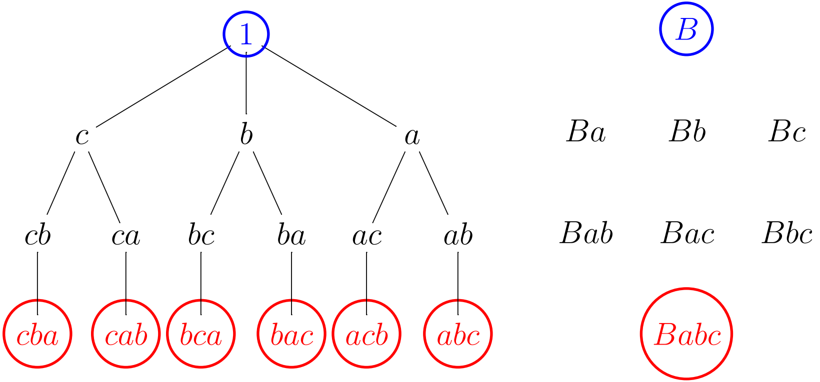

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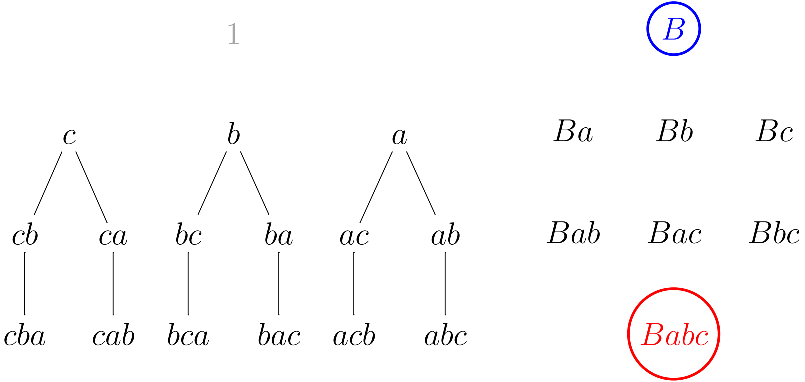

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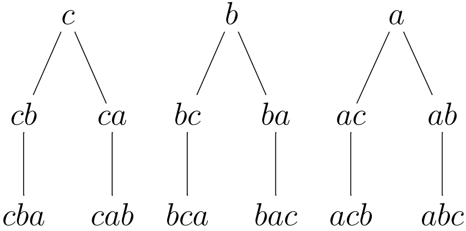

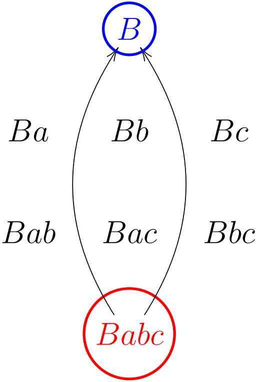

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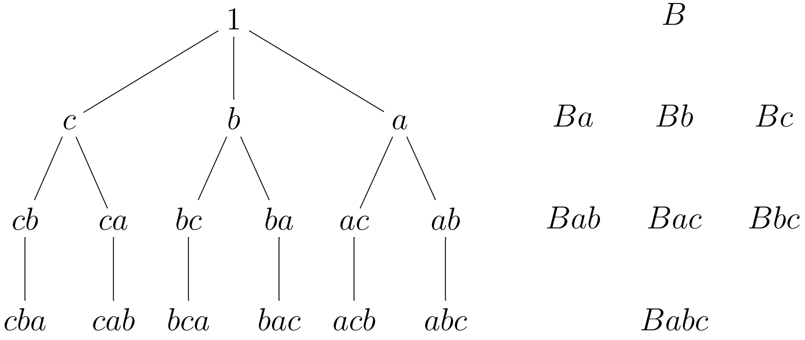

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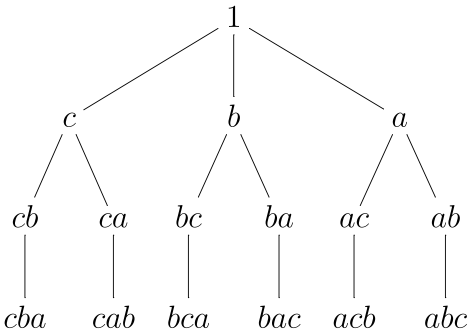

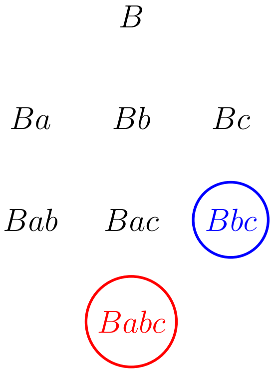

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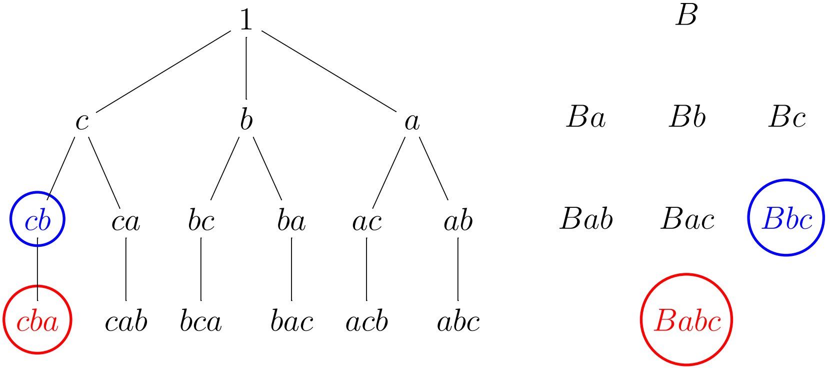

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1

$c \quad b \quad a$

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$cba \quad cab \quad bca \quad bac \quad acb \quad abc$

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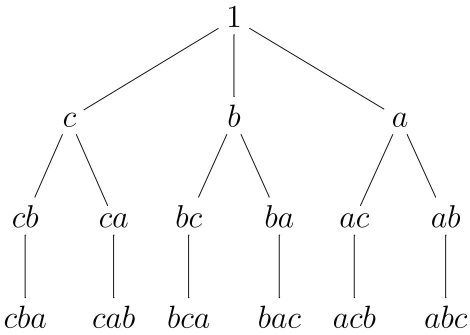

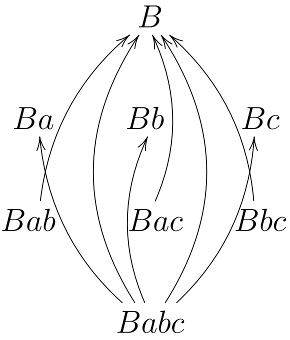

Global dimension

Let A be a finite dimensional algebra.

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- For finite-dimensional algebras, the sup can be taken over simple modules.

Global dimension and Leray numbers

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Proof outline of the Main Theorem

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This is used to compute all the spaces $\mathrm{Ext}^n(S, T)$ between simple $K(B)$ modules, S, T when K is a field and obtain the main theorem.

CW Posets and CW LRBs

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Theorem
$(P, \leq)$ is a CW poset if and only if $(P, \leq)$ is graded and for every $p \in P$, $\{q | q < p\}$ is isomorphic to a sphere of dimension $rank(p) - 1$.

Definition
An LRB B is a CW LRB if every poset $(B_X, \leq)$, $X \in \Lambda(B)$ is a CW poset.

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ranging over rank 2

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(d) *KB is a Koszul algebra and its Koszul dual is isomorphic to the dual of the incidence algebra of $\Lambda(B)$.*

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(f) Every open interval of $\Lambda(B)$ is a Cohen-Macauley poset.

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Cell Complexes, Poset Topology and the Representation Theory of Algebras Arising in Algebraic Combinatorics and Discrete Geometry

Stuart Margolis
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Representation Theory of Finite Monoids

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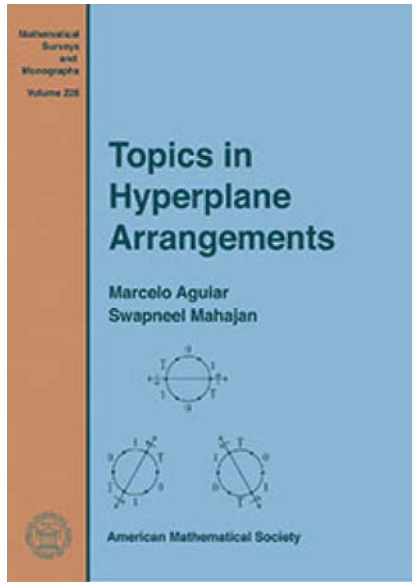
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BIMONOIDS FOR HYPERPLANE ARRANGEMENTS

Marcelo Aguiar and Swapneel Mahajan